CSIS0351: Advanced Algorithm Analysis
CSIS8601: Probabilistic Method & Randomized Algorithms

(Short name: Randomized Algorithms)

Introduction

Instructor: Hubert Chan

1 Sep 2011

Teaching Team

Instructor: Hubert Chan

Office: CYC 429

Email: hubert at cs.hku.hk

I check my email frequently!

Teaching Assistants

- Mingfei Li (HW324A, mfli at cs.hku.hk)
- Fei Chen (HW324A, fchen at cs.hku.hk)

Please use **CSIS0351** or **CSIS8601** in subject.

Why should one take this class?

- Interested in advanced algorithm design
- Learn probability tools to analyze uncertain situations
- Train analytical thinking
- Seek challenge in problem solving
- Preparation for further study

Outcomes

[Abstract Concepts] Understand abstract mathematical concepts which are involved in designing advanced algorithms, e.g., probability theory, conditional expectation, Markov's inequality, measure concentration.

[Problem Solving] Be able to apply abstract concepts in designing sophisticated algorithms to tackle various problems, e.g., MAX-CUT, MAX-3-SAT, Set Cover, PAC-learning.

[Performance Analysis] Be able to apply formal reasoning to analyze the performance of algorithms, e.g., in terms of running time, space, failure probability, number of random bits, number of samples.

[Independent and Lifelong Learning] Be inspired to learn independently and motivated to solve challenging problems.

Class Overview

Content:

- Advanced Algorithm Techniques
- Emphasis on **Probability Techniques**
- Analysis of Randomized Algorithms

Style:

- Theoretical Computer Science
- Theorems and Proofs
- Rigorous, but not pedantic
  (we assume real numbers exist!)

Teaching Style

Teaching medium: PC Tablet & white/black board

You are **strongly encouraged** to take notes!

If there is **anything** you do not understand in class, please ask!

Meeting Times

Lectures

- Mon 05:00 – 05:55PM Meng Wah Theatre 4
- Thu 03:00 – 04:55PM   Rayson Huang Theatre

Consultation Hours (1 to 1)

- Mingfei Mon 07:00 – 08:00PM HW324A
- Fei Chen Tue 05:00 – 06:00PM HW324A
- Hubert Thu 05:00 – 07:00PM CYC 429

(Please send email if you come after 6pm.)

Tutorial (group)

- Arrange as needed. (Sep 5, 7PM, CYC 308)

Tutorial

- Arrange as needed
  (No consultation hours during tutorial)
- First Tutorial on Basic Probability
  Sep 5, 7PM, CYC 308
- Please let us know if more tutorials are needed.

Timetable

Mon	Tue	Wed	Thu	Fri
5-6[Lectures] 7-8[Mingfei]	5-6[Fei]		3-5[Lectures] 5-7[Hubert]	

- Outside consultation hours: by appointments

Homepage and Newsgroup

Students are suggested to visit the course homepage regularly

Homepage:

http://www.cs.hku.hk/~c0351

http://www.cs.hku.hk/~c8601

Newsgroup: (to other students in class)

hku.cs.c0351

hku.cs.c8601

Please send email if you need a **prompt reply**.

Reference

No compulsory textbooks. Suggested extra reading:

- *Randomized Algorithms*, Motwani and Raghavan

- *The Probabilistic Method*, Alon and Spencer

Other Reference

- Class homepage in previous years 2009, 2010
- Similar classes taught in other universities
  - CMU http://www-2.cs.cmu.edu/afs/cs/academic/class/15859-f04/www
  - Washington http://www.cs.washington.edu/homes/jrl/cs525/
- Google
- Wikipedia

Assessment

3-hour examination + in-course assessment

Examination: 50%

In-course assessment: 50%

- 3~4 homeworks: 25%
- 1~2 (most likely just one) quizzes: 25%

Assessment Criteria

- Understanding is the **key point**
- Notation and equations are the means to understanding the concepts, not the ends**.**

What is understanding of a concept?
1. Ability to express in one's own words.
2. Use the concept in applicable situations.

Homeworks

Assignment Box: **CYC Building 3/F**

(Box Number confirm later)

Late Policy:

1 day late: 50% penalty

2 days late: no marks will be given

- Assignments should be submitted **before 9 pm** on the due date.

Plagiarism

Plagiarism is the action of using or copying someone else's idea or work and pretending that you thought of it or created it.

First Attempt:

Students who admit committing plagiarism for the first time shall be warned in writing and receive a zero mark for the component concerned. For those who do not confess, the case would be referred to the Programme Director for consideration.

Subsequent Attempt:

If students commit plagiarism more than once during the course of studies, the case shall be referred to the Programme Director for consideration. The Programme Director will investigate the case and consider referring it to the University Disciplinary Committee, which may impose any of the following penalties: a published reprimand, suspension of study for a period of time, fine, or expulsion from the University.

But discussing with classmates is encouraged!

Quick Survey

1. Probability
2. Random variables
3. Expectation
4. Conditional probability
5. Independence
6. Probability density function
7. Moment generating function $\phi(t) = E[e^{tX}]$
8. Chebyshev's Inequality
9. Chernoff Bound, Azuma's Inequality

First Milestone: Measure Concentration

Suppose you flip a fair coin 1000 thousands times.
How many heads do you expect to see?

The expectation of the number of heads = 500.
You would not expect to see exactly 500 heads.

However, you would be quite certain that the number of heads fall between 400 and 600.

We say that the number of heads is *concentrated around its mean*.

What is probability?

1. Sample space Ω: possible outcomes.
   Flipping a coin: $\Omega = \{ H, T \}$
2. Collection $\mathcal{F}$ of events. An event is a subset of Ω.
   (i) $\emptyset \in \mathcal{F}$
   (ii) if $A \in \mathcal{F}$, then $(\Omega \setminus A) \in \mathcal{F}$
   (iii) if $A_i \in \mathcal{F}$ for each positive integer i, then $\cup_i A_i \in \mathcal{F}$

   $\mathcal{F} = \{\emptyset, \{H\}, \{T\}, \{H,T\} \}$.

   $\emptyset$ is an impossible event, e.g., getting a "6"
3. Probability function $Pr : \mathcal{F} \rightarrow [0,1]$
   (i) $Pr(\Omega) = 1$
   (ii) For countable number of pairwise disjoint events A_i,
   $$Pr(\cup_i A_i) = \sum_i Pr(A_i).$$

Some Properties

0. If A is an event, $\overline{A}$ is the complement event $\Omega \backslash A$.
   $\Pr(\Omega \setminus A) = 1 - \Pr(A)$.
1. If A and B are events,
   $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
2. **Union Bound**. If $A_0, A_1, \ldots, A_{n-1}$ are events,
   $\Pr(\cup_i A_i) \leq \sum_i \Pr(A_i)$.

Notation: when there is no chance of confusion,
$\cup_i$ means $\cup_{i \in [n]}$
$\sum_i$ means $\sum_{i \in [n]}$

Independent Events

Two events A and B are **independent** if
$Pr(A \cap B) = Pr(A) \, Pr(B)$

Do not confuse with:
Two events A and B are **mutually exclusive** if the are disjoint, i.e., $A \cap B = \emptyset$

Conditional Probability

Suppose A and B are events such that $Pr(B) > 0$.
Then, $Pr(A \mid B) = Pr(A \cap B)/Pr(B)$.

Example.
Rolling a die.
A is the event of obtaining a number larger than 3
B is the event of getting an even number
$Pr(A \mid B) = (2/6)/(1/2) = 2/3$.

Random Variable

A random variable X is used to investigate quantities that we are interested in.
It assigns every point ω in the sample space a number X(ω).
Formally, it is a function X : $\Omega \rightarrow \mathbb{R}$.

Example. Rolling two dice. Interested in the sum.
Ω = {(i,j): 1 $\leq$ i , j $\leq$ 6}
X(i, j) = i + j

Pr(X = 0) = Pr({ω : X(ω) = 0})

Different Kinds of Random Variables

- Discrete Random Variables

  {0,1}-random variable

  Integer-valued variables
- Continuous Random Variables

Function of Random Variable

If f : $\mathbb{R} \rightarrow \mathbb{R}$ is a function and X is a random variable, then f(X) is also a random variable.

In particular, f(X) (ω) = f(X(ω))

Expectation

The expectation or mean of a random variable is

Discrete: $E[X] = \sum_{\omega} X(\omega) \Pr(\omega)$

Continuous: $E[X] = \int_{\omega} X(\omega)\, d \Pr(\omega)$

Most of the time, we focus on discrete case.

Useful equality, grouping terms (which points ω is mapped to k by X?):

$$E[X] = \sum_{\omega} X(\omega) \Pr(\omega) = \sum_{k} \sum_{\omega:\, X(\omega) = k} X(\omega) \Pr(\omega)$$
$$= \sum_{k} \sum_{\omega\,:\, X(\omega) = k} k \Pr(\omega) = \sum_{k} k \Pr(X = k)$$

Linearity of Expectation

If $X_0, X_1, \ldots, X_{n-1}$ are random variables (not necessarily independent), and $a_0, a_1, \ldots, a_{n-1}$ are numbers, then

$$E [\sum_i a_i X_i] = \sum_i a_i E[X_i]$$

Independent Random Variables

The random variables $X_0, X_1, \ldots, X_{n-1}$ are independent if for all ranges of values $I_1, I_2, \ldots, I_{n-1}$,

$$\Pr(\wedge_i X_i \in I_i) = \prod_i \Pr(X_i \in I_i)$$

Notation: $\wedge$ means "and", "intersection"

Important Fact (homework):
If X and Y are independent random variables, then $E[XY] = E[X]\,E[Y]$.

Observe this is **wrong**: $E[X^2] = (E[X])^2$