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An Approximate Approach for Area Coverage in Wireless Sensor Networks

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Abstract

In Wireless Sensor Networks (WSNs), coverage is a critical issue that has a major bearing on the quality of sensing over the target region. In this paper, we study the coverage of a region P with a transparent boundary and transparent obstacles. A transparent obstacle is an area in which a sensor cannot be deployed but through which sensing signals can pass. For cost-effectiveness, our problem is to deploy the minimum number of sensors to cover P excluding the obstacles. This problem is challenging mainly due to the fact that the target region is continuous. A straight-forward idea is to sample a finite set of crucial coverage points in P, thus making the coverage space discrete. Most existing approaches, however, tend to either require too many sampled points, which leads to increased running time, or have an inferior coverage of the region.

We propose a discretization approach which converts the area coverage problem into the problem of Minimum Geometric Disk Cover with Candidate Positions (MGDCCP) which is proved to be strongly NP-hard. We present a polynomial-time approximation scheme (PTAS) based on the shifting strategy for the MGDCCP problem. Specifically, our approach guarantees covering a $(1 - \varepsilon)$ fraction of the region with probability no less than $(1 - \frac{\varepsilon}{h})$ using at most $(1 + \frac{1}{l})^2 h$ sensors, where h is the theoretical minimal number of sensors needed to cover the region P, l is a positive integer parameter in the shifting strategy, and $\varepsilon \in (0, 1)$ is the covering tolerance. Furthermore, we show that our proposed approach is output-sensitive with time complexity that is polynomial in the input size and the optimal solution size. Therefore, for any fixed parameter l and ε, the coverage accuracy, the running time, the approximation ratio and the success probability are all bounded.

Keywords: Area Coverage, ε-net, Random Sampling, Shifting Strategy, Wireless Sensor Networks

1. Introduction

Wireless sensor networks (WSNs) have sparked much research interests in recent years due to their extensive application in military as well as in many civilian domains such as health care, environmental protection, intrusion detection, cancer monitoring, and smart agriculture [1]. Of the many issues in deploying WSN, coverage is among the most fundamental ones, of which a sufficiently high degree is necessary for the intended service to be satisfactorily provided.

According to the application scenario, sensor deployment can be classified into two kinds, random deployment and deterministic deployment. Random deployment is widely used in large-scale outdoor monitoring, such as battle fields, oceans, and forests, where the target area is usually unfriendly to human beings and environmental information is hard to obtain ahead of deployment. Deterministic approaches are used to meet specific coverage requirements with the minimum number of sensors. Common examples include residential area monitoring and medical applications, where sensors equipped with cameras are installed in carefully selected spots. In contrast to the random deployment, the coverage of deterministic methods is a function of the exact locations of the sensors and not the density of the sensors. The solutions we present in this paper are applicable to deterministic approaches with complete pre-knowledge of the target region.

A classic example of coverage is the one by Kershner [3] who, in 1939, studied the well-known problem of deploying the minimum number of disks to cover a region based on a tessellation of regular hexagons; he proved that it is asymptotically optimal to place disks in the hexagon pattern when the disk radius r is small enough with respect to the whole region. For WSNs, Bai et al. [4] presented several deployment patterns to achieve full coverage in a plane when taking network connectivity into account. Other researchers have studied the coverage problem in a finite region with obstacles through computational geometry [5, 6, 7]. The authors of [5] proposed a heuristic algorithm to fully cover a region having arbitrary (opaque) obstacles, which allow neither the sensor to be placed inside nor the signals to pass through. They first deploy an optimal pattern for covering a plane over the region, and then locate and efficiently cover the uncovered holes formed by the obstacles. In [6], the authors proposed an algorithm that firstly deploys sensors with distance $\sqrt{r_s}$ along the boundaries, and then Delaunay triangulation is applied to cover the rest of the region. In [7], the whole region is divided into single-row and multi-row sub-regions; the multi-row regions are covered with some deployment patterns and each single-row region is covered by a line of sensors greedily. Recently, random sampling has been frequently used to discretize the continuous coverage problem. The authors of [8] presented a Sampling Theorem utilizing VC-dimension and ε-net. Based on this Sampling Theorem, the authors of [9] studied the number of sensors required to cover a sufficiently large fraction of an area by random deployment. Agarwal et al. in [2] applied the Sampling Theorem to the model of a finite region with obstacles. They presented an approximation algorithm to cover a large fraction of the region with high probability given a sampling of the points to be covered.

Because of the wide range of practical applications and their requirements, abundant research results exist on the coverage problem in various models, such as the coverage of directional cameras [13], the coverage with mobile sensors [10, 11] and the coverage with minimum energy [14]. We have mentioned only the ones mostly related to our work. More results can be founded in recent surveys (e.g., [15]).

Our Contribution: In this paper, we focus on deploying the minimum number of sensors to cover a region with a transparent boundary and transparent obstacles (refer to Section 2.1). Our method comprises two main components:

- we randomly sample a finite set of points, called *landmarks*, and show that most of the area will be covered with high probability by covering these landmarks according to the Sampling Theorem;
- we present a polynomial-time approximation scheme (PTAS) to cover the landmarks based on a shifting strategy.

Precisely, we derive a $(1 + \frac{1}{l})^2$-approximation algorithm that can cover a $(1 - \varepsilon)$ fraction of the region with probability no less than $(1 - \frac{\varepsilon}{h})$, where l is a positive integer parameter in the shifting strategy and $0 < \varepsilon < 1$. Our algorithm is output-sensitive with time complexity that is polynomial in the input size and the optimal solution size. For any fixed parameter l and ε, the coverage accuracy, running time, approximation ratio and success probability are all bounded. Thus we can ensure the performance of the algorithm in all respects.

Paper Organization: The rest of this paper is organized as follows. In Section 2, we introduce the preliminaries and give the formal definitions of the problem. In Section 3 we modify Agarwal et al.'s algorithm which is based on the Sampling Theorem, and derive the landmark-based algorithm for our prolbem. In Section 4 we present a PTAS to cover the landmarks. Section 5 is devoted to an analysis of our approach. Section 6 concludes the paper and suggests some open problems and future directions.

2. Problem Definitions and Preliminaries

2.1. Models and Problem Definitions

The region of interest is a finite 2D area with an arbitrary transparent boundary and arbitrary transparent obstacles (Figure 1(a)). A transparent obstacle is a special area (sub-region) in which sensors can not be placed but the sensing signals can penetrate through it. In practice, transparent obstacles can be quite common, e.g., an outdoor pool, busy corridors in an office building, a city section with many private properties. Both the region and the obstacles are modeled as simple polygons in the 2D plane. We denote the area in the region excluding the obstacles as P, which needs to be covered by sensors. We assume sensors are stationary after installation, and they are homogeneous—they all have a fixed sensing radius, r_s. The binary sensor model is adopted: the sensing range of sensor s located at a point $x \in P$ is a disk centered at x with radius r_s. As there are only transparent obstacles, a point $p \in P$ is covered by s if and only if it is within a distance of r_s from s. Therefore, for the sensor located at x, its sensing region is $V(x) = \{p \in P| \ \|px\| \leq r_s, p \in P\}$ (Figure 1(b)). We say a region P is covered if every point $p \in P$ is covered by at least one sensor.

(a) (b)

Fig. 1: Model Definition: (a) A polygon region P with transparent obstacles; (b) coverage of a sensor: the sensor S covers the yellowish area, where r_s is the sensing radius.

Thus, our problem can be presented as follows: given a region with a transparent boundary and possibly transparent obstacles, place a minimum number of sensors to meet the coverage requirement, i.e., to cover $(1 - \varepsilon)$ fraction of the entire region P, where $0 < \varepsilon < 1$ is a constant.

2.2. *ε-net and VC-dimension*

In contrast to the minimum geometric disk cover problem (MGDC), which is to place the minimum number of disks to cover some finite discrete points on the 2D plane [12], we study the coverage of a 2D region. We adopt the widely used method in computational geometry: sampling finite points, so-called *landmarks*, such that it suffices to cover just these landmarks in order to guarantee a coverage of most of the given region. Intuitively, the more points we sample, the more precise our result can be, but the worse the time complexity. To analyze the trade-off between accuracy and running time, we introduce some concepts from uniform random sampling and statistical learning theory: ε-net and VC-dimension [8].

A *range space* $(X, \mathcal{R})$ is a set X along with a collection $\mathcal{R}$ of subsets of X; these subsets are called ranges.

Definition 1 (ε-net)**.** *For a given $\varepsilon > 0$, a subset $N \subseteq X$ is called an ε-net of a range space $(X, \mathcal{R})$ if $r \bigcap N \neq \emptyset$ for all $r \in \mathcal{R}$ such that $|r| \geq \varepsilon|X|$.*

In other words, an ε-net N hits all large enough sets in $\mathcal{R}$. To get a preliminary idea of what is to come, we can regard the ε-net as the landmarks that need to be covered in our algorithm. We show in the next section that covering the ε-net is sufficient for covering most of P.

We now introduce another important concept, VC-dimension, the bridge between the uniform random sampling points in P and the ε-net.

Definition 2 (VC-dimension)**.** *Given a range space $(X, \mathcal{R})$, let A be a subset of X. We say A is shattered by $\mathcal{R}$ if for all $Y \subseteq A$, $\exists r \in \mathcal{R}$ such that $r \cap A = Y$. The VC-dimension of $(X, \mathcal{R})$, denoted by VC-dim($(X, \mathcal{R})$), is the cardinality of the largest set that can be shattered by $\mathcal{R}$.*

2.3. *Sampling Theorem*

In this section, we introduce and modify the sampling theorem in [2] according to our specific coverage problem so that we can convert area coverage to discrete point coverage.

Set $k = c_1 h$, where h is the minimum number of sensors that cover the region P and $c_1 > 1$ is a constant. We define a range space $\Sigma_k = (P, \overline{C}_k)$, where each range in $\overline{C}_k$ is the complement of the union of the coverage areas of at most k sensors in P. Assume L to be an ε-net of Σ_k. As $L \subseteq P$, there must be a set S of sensors with $|S| \leq k$ that covers L, i.e., $L \subseteq V(S)$. Then, we have $L \bigcap (P \setminus V(S)) = \emptyset$. As L is an ε-net, $|P \setminus V(S)| \leq \varepsilon |P|$. That is, S covers at least $(1-\varepsilon)$ fraction of the area P.

We then use random sampling to get the ε-net L in range space $\Sigma_k = (P, \overline{C}_k)$. It is known that a disk on the plane has VC-dimension 3. In our problem, as there are only transparent obstacles which need not be covered, for one sensor, VC-dim(Σ_1) is also 3. The VC-dimension of a union of k concepts formed by constant-dimensional concepts is $O(k \log k)$ [16], so VC-dim(Σ_k) = $O(k \log k)$. According to the result in [8], for a range space $(X, \mathcal{R})$ with VC-dimension d, a random subset $N \subseteq X$ of size $\Omega((d/\varepsilon)\log(d/(\varepsilon\delta)))$ is an ε-net of $(X, \mathcal{R})$ with probability at least $1 - \delta$.Therefore, we can present the Sampling Theorem as used in our problem by setting $\delta = \varepsilon/h$.

Theorem 1 (Sampling Theorem)**.** *Let P be a polygon region with a transparent polygonal boundary in $\mathbb{R}^2$ and with or without transparent obstacles. Let $\varepsilon > 0$ be a parameter. Suppose P can be covered by h sensors. Let $L \subset P$ be a random subset of $m = c_2 \frac{h \log h}{\varepsilon} \log \frac{h^2 \log h}{\varepsilon^2}$ points in P, and let S be a set of at most $k = c_1 h$ sensors that covers L, where $c_1, c_2 \geq 1$ are sufficiently large constants. Then S covers $(1-\varepsilon)$ fraction of the area of P with probability at least $1 - \varepsilon/h$.*

3. Landmark-Based Algorithm

In this section, we introduce a landmark-based algorithm to solve the coverage problem according to Agarwal et al.'s method in [2]. If there is an oracle that gives the minimum number of sensors h for full coverage of P, we can get the ε-net with high probability by the Sampling Theorem. The problem is then converted to its discrete version, MGDC, where the points needed to be covered are the landmarks. Let S be a set of sensors that cover L. We can guarantee that S can cover $(1-\varepsilon)$ fraction of P with high probability.

However, there is no such an oracle for obtaining the optimal h. Based on Agarwal et al.'s algorithm, we can guess the value of h from 1 to c^i until h is assigned a large enough value. $c > 1$ is a constant parameter here. Recall that when h is large enough, the probability that L is an ε-net is high, which we can use as the verifier of the value of h. There are two requirements for an ε-net L produced based on the Sampling Theorem: 1) k sensors are enough to cover L, and 2) any k sensors that cover L will cover at least a $(1-\varepsilon)$ fraction of P. Here $k = c_1 h$ means the number of sensors that cover the set of landmarks L. We rewrite Agarwal et al.'s algorithm with our parameters for the Sampling Theorem as Algorithm 1. Figure 2 illustrates the landmarks computed for the region P.

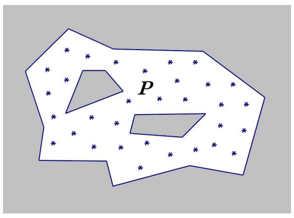


Fig. 2: The landmarks to be covered in P: the blue stars

In Line 8, we assume there is an algorithm MGDC(P, L) which gives a solution S to cover the landmarks L. In Agarwal et al.'s algorithm, MGDC(P, L) is a simple greedy method L [2]. In the next section, we present an $(1 + \frac{1}{l})^2$-approximation algorithm, MGDC(P,L), which is based on a shifting strategy to cover L.

4. Algorithm For Minimum Geometric Disk Cover Problem

By randomly sampling the landmarks, we convert the original coverage problem into its discrete version, the Minimum Geometric Disk Cover Problem (MGDC). MGDC is known to be NP-complete and there is no *fully polynomial approximation scheme* for the problem, unless P=NP [12].

In this section we present an approximation algorithm that computes the placement of sensors to cover the landmarks L in Algorithm 1. We first introduce an efficient strategy to find all the candidate positions for the sensors.We then propose an approximation algorithm the runtime of which is polynomial in the number of vertices of P as well as the obstacles and the number of landmarks L.

Algorithm 1 SensorDeployment (P, ε)

INPUT: Polygon region P, error threshold ε
OUTPUT: A placement of sensors covering $(1-\varepsilon)|P|$.
$i := 1$
repeat
  $h' := c^i$,   */*try the value h from 1 to c^i */*
  $k := c_1 h'$, $m := c_2 \frac{h' \log h'}{\varepsilon} \log \frac{h'^2 \log h'}{\varepsilon^2}$
  $L := m$ random points in P
  $S :=$ MGDC(P, L)
  $i := i + 1$
until $(|S| \le k)$ and $(|V(S)| = |\bigcup_{s \in S} V(s)| \ge ((1-\varepsilon)|P|)$
return S

4.1. Finding Candidate Locations for the Sensors

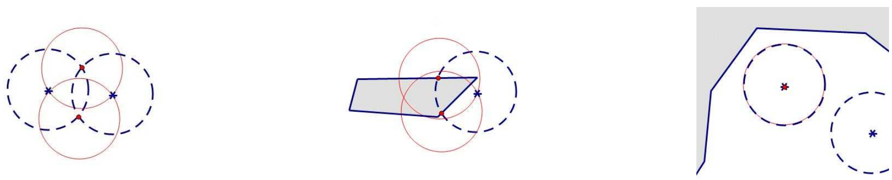
In this section, we present a strategy to find a finite set of candidate positions for sensors to make MGDC computationally solvable. We first define basic sensor locations as follows.

Definition 3 (Basic Sensor Location)**.** *Given a region of interest P and a set of landmarks, a basic sensor location is a position in P that satisfies at least one of the following three conditions when a sensor is deployed to the position.*

I. *There are at least two landmarks on the sensing boundary (Figure 3(a)).*

II. *There is at least one landmark on the sensing boundary and the sensor is located exactly on the boundary of an obstacle or the region P (Figure 3(b)).*

III. *A landmark is* isolated *if and only if the distance from its closest neighboring landmark is larger than* $2r_s$ *and there is no obstacle (nor the region boundary) that is within a distance of* r_s *from the landmark. For an isolated landmark, the landmark's position is called a basic sensor location (Figure 3(c)).*



(a) (b) (c)

Fig. 3: Three types of basic locations for the sensors. The stars are the landmarks. The red dots represent basic sensor locations generated according to the landmarks.

The basic sensor locations are denoted as a set C, which are used as the candidate positions of sensors to cover the landmarks (Figure 4). In addition, we define the Minimum Geometric Disk Cover with Candidate Positions problem (MGDCCP(P,L)) as: given a set of landmarks L in the region P, find the minimum number of sensors deployed at the basic locations to cover all the landmarks. Note that MGDCCP is a special case of the set cover problem, i.e., we can treat the landmarks as elements, the coverage range of a sensor located at a candidate position as a set, and define an element to be in a set as long as the landmark is covered by the sensor. Further, we have the following theorem.

Theorem 2. *For a fixed region P and the set of landmarks L, MGDCCP(P, L) and MGDC(P, L) have optimal solutions of the same size. Furthermore, an optimal solution of MGDCCP(P, L) is indeed an optimal solution of MGDC(P, L).*

Proof. We prove the theorem by showing that 1) a feasible solution of MGDCCP(P, L) is also a feasible solution of MGDC(P, L); 2) given a feasible solution of MGDC(P, L), we can find a feasible solution of MGDCCP(P, L) with a smaller or equal size.

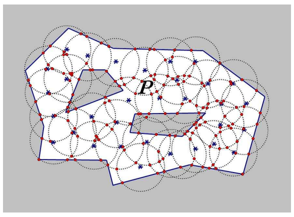


Fig. 4: Candidate locations for the sensors in MGDCCP. Red points are the candidate locations, and the blue stars are the landmarks to be covered.

The first part of the proof is trivial: Since the candidate positions for sensors are in P, any feasible solution F' of MGDCC(P,L) that covers all the landmarks must be a feasible solution of MGDC(P,L).

Then the opposite direction. The coverage set of a sensor s located at point $p \in P$ is defined as the set of landmarks covered by s, denoted as $S(p)$. A sensor can be moved freely without losing an existing element in its coverage set until its sensing boundary hits a landmark in $S(p)$. Thus, for any sensor s located at $p \in P$ that covers at least one landmark, it can always be moved to a basic sensor position p' with a coverage set $S(p) \subseteq S(p')$. For any feasible solution F of MGDC(P,L) with sensor locations $\{p_1, p_2, \ldots, p_t\}$, sensors that do not cover any landmark can be omitted at the beginning. Thus, we get a feasible solution of MGDC(P,L) with sensor locations $\{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_{t'}\}$ of a smaller or equal size. There exists a feasible solution with sensor locations $\{p'_1, p'_2, \ldots, p'_{t'}\}$ of MGDCCP(P,L), such that $S(\tilde{p}_i) \subseteq S(p'_i)$ for all $i \in \{1 \cdots t'\}$. So for any feasible solution of MGDC(P,L), a feasible solution of MGDCCP(P,L) with a smaller or equal size can be found.

Combining the two parts above, the theorem is proved. □

For the set of landmarks L ($|L| = m$), the number of the candidate positions that satisfy condition I is $O(m^2)$. As for condition II, the candidates are the intersection of the boundary of P and the circle with the center on the landmarks. This implies the number of candidates of sensor placement for condition II is $O(mn)$, where n is the number of vertices of the region P and obstacles. The number of candidates for condition III is $O(m)$. Thus, the total number of candidates is $|C| = O(m^2 + mn)$.

In conclusion, the number of candidate positions for sensors is finite and can be computed in $O(m^2+mn)$ time. Thus we can solve MGDC by solving MGDCCP.

4.2. *Approximation Algorithm for Minimum Geometric Disk Cover with Candidate Positions*

To solve MGDC, we first compute the basic sensor locations and covert the problem to the MGDCCP problem. As mentioned before, MGDCPP is a special case of the Set Cover Problem. Using the same algorithm to solve the Set Cover Problem, a greedy one for instance, we can have an $ln|m|$ approximation. Fortunately, due to the geometric property, we can get a better result for MGDCCP through the shifting strategy as in [12].

The basic idea is divide-and-conquer. Firstly, we divide the area P into small enough grids so that we can solve the problem for each grid by using a brute-force algorithm. Then we combine the results to obtain an approximation. We consider all the possible *shifts* of the grids and maintain the best approximation.

Let $D = 2r_s$ be the diameter of the disk. We first divide the area P into vertical strips of width D. For a shifting parameter l (a positive integer), we make l consecutive strips as a group with width $l \times D$. There are l possible ways for the grouping because of the l different start positions for the leftmost group of width $l \times D$. We denote the partitions as Par^V_1,$Par^V_2, \ldots, Par^V_l$ according to the start positions. Similarly, we can partition the area P horizontally, and the horizontal partitions are denoted as Par^H_1,$Par^H_2, \ldots, Par^H_l$. So, we get a set of grids $Par_{i,j}$ by combining vertical partitions Par^V_i and horizontal partitions Par^H_j for all $i, j \in \{1 \cdots l\}$.

Let A be any local algorithm for MGDCCP with approximation ratio r_A. $A(Par^V_i)$ denotes the algorithm that applies A to each member of the strip group in a given vertical partition Par^V_i and outputs the union of all disks. Maintaining the minimum answer among all the possible partitions Par^V_i, we get the approximation ratio, denoted as $r_{gen(A)}$, as in Lemma 3 [12].

Lemma 3 (Shifting Lemma)**.** *For a given shifting parameter l, the approximation ratio of the shifting algorithm on the vertical strips is* $r_{gen(A)} \leq r_A(1 + \frac{1}{l})$.

Algorithm 2 MGDC(P, L): Approximation Algorithm for MGDC:

Input: Polygon area P, the landmarks and candidates in P, shifting parameter l and sensing range r_s.
Output: a subset of P which can cover all the landmarks
Divide P into vertical and horizontal strips with width D
Set S := {all candidates for sensors}
for $i := 1$ **to** l **do**
  for $j := 1$ **to** l **do**
    Generate shifting partition $Par_{i,j}$; Set $S' := \emptyset$
    for every grid g in $Par_{i,j}$ **do**
      compute all the candidates of landmarks in grid g
      T = candidates chosen to cover all the landmarks in grid g
      $S' = S' \cup T$
    if $|S| > |S'|$ **then** $S = S'$
return S

In our approximation method for MGDC(P,L) (Algorithm 2), A is a brute-force algorithm with approximation ratio 1. We use the shifting lemma twice, one for the vertical and one for the horizontal. So the algorithm gives a $(1 + \frac{1}{l})^2$-approximation.

In a partition of P, let c_k and m_k be the number of candidates and landmarks, respectively, in a grid g_k. Here we have $\sum_{g_k} m_k = m$ and $c_k = O(m_k n + m_k^2)$. The grid g_k can always be covered by $(\sqrt{2}l)^2 = 2l^2$ sensors, independent of the number of landmarks. Since we have an upper bound for the optimal solution in the grid, we can reduce the number of possible solutions tried by the brute-force algorithm from $O(2^{c_k})$ to $O(c_k^{2l^2})$. Verifying each of the assignments in a grid takes time $O(m_k l^2)$. For each partition the running time is $\sum_{\forall\ grid\ g_k} O(c_k^{2l^2} m_k l^2) = O((m^2 + mn)^{2l^2} ml^2)$. There are l^2 partitions in total. Thus, the running time for the algorithm is $O((m^2 + mn)^{2l^2} ml^4))$. Figure 5 gives an example to illustrate the locations of sensors and the area covered as generated by our algorithm.

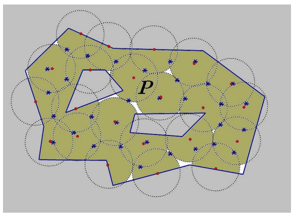


Fig. 5: Coverage of the region P: the red dots are the sensors deployed; the colored area in P is covered while the white areas are the uncovered holes.

5. Performance Analysis

As the performance of a deployment approach is extremely dependent on the specific target region, i.e., the numbers, positions and shapes of obstacles, simulations for a limited number of specific artificial cases might not give significant evidence of the performance. Therefore, in this paper, we focus on the theoretical analysis of our deployment algorithm (Algorithm 1), in which Algorithm 2 is adopted to compute $MGDC(P, L)$. The analysis in the following can serve as a basis for simulation studies in the future.

Correctness: Suppose the algorithm halts when $h' = c^i$. It is obvious that the output set S satisfies the second requirement of ε-net, $|V(S)| \geq (1 - \varepsilon)|P|$. This means S is a valid deployment for sensors that covers $(1 - \varepsilon)$ fraction of the area.

Approximation Ratio: From the analysis in Section 4.2, the approximation ratio of MGDC(P,L) is $(1+\frac{1}{l})^2$. Since h is the minimal number of sensors to cover P and $L \subseteq P$, h sensors are also enough to cover L, which implies that $|S| \leq (1 + \frac{1}{l})^2$ OPT-MGDC(P,L)$\leq (1 + \frac{1}{l})^2 h = O(h)$, where OPT-MGDC($P$,$L$) is the optimal solution of MGDC(P,L). Thus, with a probability of $(1 - \frac{\varepsilon}{h})$, our algorithm gives a solution with size no larger than $(1 + \frac{1}{l})^2 \times h$ to cover a $(1 - \varepsilon)$ fraction of P. The approximation ratio of Algorithm 1 is $(1 + \frac{1}{l})^2$.

Time Complexity: For any output S of Algorithm 1, we say that S is *effective* if $|S| \leq (1 + \frac{1}{l})^2 h$ for a given l in the shifting strategy. Set $c_1 = (1 + \frac{1}{l})^2$. According to the Sampling Theorem, the probability of the landmarks consisting of an ε-net increases with the increasing of h'. When $h' > h$, we have $k = (1 + \frac{1}{l})^2 h$. From Algorithm 1, the first requirement of ε-net, $|S| \leq k$, is always satisfied. For the second requirement, when $c^{i-1} < h \leq c^i = h'$, the algorithm halts with probability $1 - \varepsilon/h' > 1 - \varepsilon/h$, and we consider the running time when it halts here.

The algorithm runs $O(log_c h)$ rounds for MGDC(P, L_i) before $h' < ch$, where L_i is the ε-net in i^{th} round. Note that the algorithm may halt before that and gives an effective solution. In the i^{th} round before $h' > ch$, the running time of MGDC(P,L) is $O((mn + m^2)^{2l^2} ml^4)$, where $m = c_2 \frac{h \log h}{\varepsilon} \log \frac{h^2 \log h}{\varepsilon^2}$ is polynomial in h.

The running time of Algorithm 1 before $h' > ch$ is $O(((mn + m^2)^{2l^2} ml^4) log_c h)$, where m is the number of the landmarks in the last round and polynomial in h.

In conclusion, with probability no less than $1 - \frac{\varepsilon}{h}$, the algorithm will halt within time polynomial in the input size and the output size and give an effective solution. For any fixed parameter l and ε, the coverage accuracy, running time, approximation ratio and success probability are all bounded, which indicates that the performance of our algorithm is guaranteed in all respects.

6. Conclusion and Future Work

Coverage is an important aspect in the design and deployment of wireless sensor networks. In this paper, we study the problem of deterministically deploying the minimum number of sensors to cover a region with a transparent boundary and transparent obstacles. Based on the Sampling Theorem, we find a set of landmarks by covering which we can achieve coverage of most of the region with high probability. Then based on the shifting strategy, we propose a polynomial-time approximation scheme to cover the landmarks. For a fixed approximation ratio, our algorithm will halt within time polynomial in both the input and output size with high probability. Future work can be carried out in many directions. It is interesting to propose more efficient algorithms with better approximation for some special cases, such as when the region and obstacles are of special shapes. We can also take the connectivity into account in the deployment where there are obstacles. 3D coverage with obstacles is a also meaningful direction for future work.

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