Deterministic Online Call Control in Cellular Networks and Triangle-Free Cellular Networks

Joseph Wun-Tat Chan[1], Francis Y.L. Chin[2⋆], Xin Han[3], Ka-Cheong Lam[2,4], Hing-Fung Ting[2⋆⋆], and Yong Zhang[2]

[1] College of International Education, Hong Kong Baptist University, Hong Kong.
cswtchan@gmail.com
[2] Department of Computer Science, The University of Hong Kong, Hong Kong
{chin,hfting,yzhang}@cs.hku.hk
[3] School of Software, Dalian University of Technology, China
hanxin.mail@gmail.com
[4] College of Computer Science, Zhejiang University, China
pandaman@163.com

Abstract. Wireless Communication Networks based on Frequency Division Multiplexing (FDM in short) plays an important role in the field of communications, in which each request can be satisfied by assigning a frequency. To avoid interference, each assigned frequency must be different to the neighboring assigned frequencies. Since frequency is a scarce resource, the main problem in wireless networks is how to utilize the frequency as fully as possible. In this paper, we consider the call control problem. Given a fixed bandwidth of frequencies and a sequence of communication requests, in handling each request, we must immediately choose an available frequency to accept (or reject) it. The objective of call control problem is to maximize the number of accepted requests. We study the asymptotic performance, i.e., the number of requests in the sequence and the number of available frequencies are very large positive integers. In this paper, we give a 7/3-competitive algorithm for call control problem in cellular network, improving the previous 2.5-competitive result. Moreover, we investigate the triangle-free cellular network, propose a 9/4-competitive algorithm and prove that the lower bound of competitive ratio is at least 5/3.

1 Introduction

Frequency Division Multiplexing (FDM in short) is commonly used in wireless communications. To implement FDM, the wireless network is partitioned into small regions (cell) and each cell is equipped with a base station. When a call request arrives at a cell, the base station in this cell will assign a frequency to this request, and the call is established via this frequency. Since frequency is a

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scarce resource, to satisfy the requests from many users, we have to reuse the same frequency for different call requests. But if two neighboring calls are using the same frequency, interference will appear to violate the quality of communications. Thus, to avoid interference, the same frequency cannot be assigned to two different calls with distance close to each other. In general, the same frequency cannot be assigned to two calls in the same cell or neighboring cells.

There are two research directions on the fully utilization of the frequencies. One is *frequency assignment problem*, and the other is *call control problem.* In frequency assignment problem, each call request must be accepted, and the objective is to minimize the number of frequencies to satisfy all requests. In call control problem, the bandwidth of frequency is fixed, thus, when the number of call requests in a cell or in some neighboring cells is larger than the total bandwidth, the request sequence cannot be totally accepted, i.e., some requests would be rejected. The objective of call control problem is to accept the requests as many as possible.

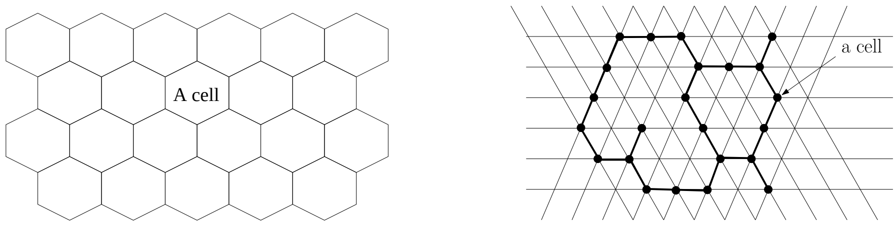
Problem Statement:

In this paper, we consider the online version of call control problem. There are ω frequencies available in the wireless networks. A sequence σ of call requests arrives over time, where $\sigma = \{r_1, r_2, ..., r_t, ...\}$, c_t denote the t-th call request and also represent the cell where the t-th request arrives. When a request arrives at a cell, the system must either choose a frequency to satisfy this request without interference with other assigned frequencies in this cell and its neighboring cells, or reject this request. When handling a request, the system does not know any information about future call requests. We assume that when a frequency is assigned to a call, this call will never terminate and the frequency cannot be changed. The objective of this problem is to maximize the number of accepted requests.

We focus on the call control problem in cellular networks and triangle-free cellular networks. In the cellular network, each cell is a hexagonal region and has six neighbors, as shown in Figure 1(a). The cellular network is widely used in wireless communication networks. A network is *triangle-free* if there are no 3-cliques in the network, i.e., there are no three mutually-adjacent cells. An example of a triangle-free cellular network is shown in Fig. 1(b).

Performance Measure:

To measure the performance of online algorithms, we often use the competitive ratio, which compare the output between the online algorithm and the optimal offline algorithm, which knows the whole request sequence in advance. In call control problem, the output is the number of accepted requests. For a request sequence σ, let $A(\sigma)$ and $O(\sigma)$ denote the number of accepted request of an online algorithm A and the optimal offline algorithm O, respectively. The competitive ratio of algorithm A is $R_A = \sup_\sigma O(\sigma)/A(\sigma)$. For the call control problem, we focus on the asymptotic performance, i.e., the number of requests and the number of frequencies are large positive integers. The asymptotic com-



(a) cellular network (b) triangle-free cellular network

Fig. 1. An example of the cellular network and triangle-free cellular network

petitive ratio for an online algorithm A is

$$R_A^\infty = \limsup_{n \to \infty} \max_\sigma \{ \frac{O(\sigma)}{A(\sigma)} | O(\sigma) = n \}.$$

Related Works:

How to fully utilize the frequencies to satisfy the communication requests is a very fundamental problem in theoretical computer science and engineering. Both the frequency assignment problem and the call control problem are well studied during these years. From the description of these two problems, we know that the call control problem is the dual problem of the frequency assignment problem.

The offline version of the frequency assignment problem in cellular networks was proved to be NP-hard by McDiarmid and Reed [6], and two 4/3-approximation algorithms were given in [6, 7]. For the online frequency assignment problem, when a call request arrives, the network must immediately assign a frequency to this call without any interference. There are mainly three strategies: Fixed Allocation [5], Greedy Assignment [1], and Hybrid Assignment [3]. If the duration of each call is infinity and the assigned frequency cannot be changed, the hybrid algorithm gave the best result for online frequency assignment, i.e., a 2-competitive algorithm for the absolute performance and a 1.9126-competitive algorithm for the asymptotic performance. When the background network is triangle-free, a 2-local 5/4-competitive algorithm was given in [9], an inductive proof for the 7/6 ratio was reported in [4], where k-local means when assigning a frequency, the base station only knows the information of its neighboring cells within distance k. In [11], a 1-local 4/3-competitive algorithm was given.

For the call control problem, the offline version is NP-hard too [6]. To handle such problem, greedy strategy is always the first try, when a call request arrives, the network choose the minimal available frequency to serve this request, if any frequency is interfere with some neighboring assigned frequency, the request will be rejected. Pantziou et al. [8] analyzed the performance of the greedy strategy, proved that the asymptotic competitive ratio of the greedy strategy is equal to the maximal degree of the network. Caragiannis et al. [1] gave a randomized algorithm for the call control problem in cellular networks, the asymptotic competitive ratio of their algorithm is 2.651. Later, the performance of the randomized algorithms was improved to 16/7 by the same authors [2], they also proved the lower bound of the asymptotic competitive ratio for the randomized algorithm is at least 2. Very recently, a deterministic algorithm with asymptotic competitive ratio 2.5 was given in [10], and the lower bound of the asymptotic competitive ratio for the deterministic algorithm was proved to be 2.

Our Contributions:

In this paper, we consider the deterministic algorithms for the call control problem in cellular networks and triangle-free cellular networks. In cellular network, we give a 7/3-competitive algorithm, improving the previous 2.5-competitive result. In triangle-free network, we propose a 9/4-competitive algorithm, moreover, the lower bound of the competitive ratio in triangle-free network is proved to be at least 5/3.

2 Call Control in Cellular Networks

2.1 Algorithm

The idea of our algorithm for call control problem in cellular networks is similar to the algorithm in [10]. By using a totally different analysis, we can show our algorithm is better, moreover, our algorithm is best possible among this kind of algorithms.

Cellular networks is 3 colorable, each cell can be associated with a color from $\{R, G, B\}$ and any two neighboring cells are with different colors. Partition the frequencies into four sets, F_R, F_B, F_G, and F_S, where F_X $(X \in \{R, G, B\})$ can be only used in cells with color X and F_S can be used in any cell. Since we consider the asymptotic performance of the call control problem, we may regard the number ω of frequencies in the system is a multiple of 7. Divide the the frequencies into four disjoint set as follows:

$$F_R = \{1, ..., 2\omega/7\},$$
$$F_G = \{2\omega/7 + 1, ..., 4\omega/7\},$$
$$F_B = \{4\omega/7 + 1, ..., 6\omega/7\}, and$$
$$F_S = \{6\omega/7 + 1, ..., \omega\}$$

Obviously, the ratio between the number of frequencies in F_R, F_G, F_B, and F_S is 2 : 2 : 2 : 1.

Now we describe our algorithm CACO as follows:

2.2 Analysis

The high level idea to prove the performance of our algorithm CACO is to show that the ratio between the total number of accepted requests by CACO and the

Algorithm 1 CACO : When a request arrives at a cell C with color $c \in \{R, G, B\}$

1: **if** F_c is not totally used up **then**
2: assign the minimal available frequency from F_c to satisfy this request.
3: **else if** F_S is not totally used up in cell C and its neighboring cells **then**
4: assign the minimal available frequency from F_S to satisfy this request.
5: **else**
6: reject this request.
7: **end if**

total number of satisfied requests by the optimal offline algorithm is at least 3/7. To prove this, we analyze the number of satisfied requests in each cell and its neighboring cells, then compare the number with the optimum value.

Let R_i be the number of the requests arrived at cell C_i. Let O_i be the number of requests accepted by the optimal offline algorithm in cell C_i. $\sum O_i$ is the total number of accepted request by the optimal offline algorithm. Let A_i be the number of requests accepted by our online algorithm CACO in cell C_i. $\sum A_i$ is the total number of accepted request by CACO. Let $G_x(C_i)$ be the the number of requests accepted by CACO in cell C_i by assigning frequencies from F_x. It can be seen that $A_i = G_R(C_i) + G_G(C_i) + G_B(C_i) + G_S(C_i)$. If C_i is colored with $x \in \{R, G, B\}$, then $A_i = G_x(C_i) + G_S(C_i)$.

Fact 1 *For each cell C_i, $O_i \leq R_i$, $A_i \leq R_i$, and $A_i \geq 2\omega/7$ when $R_i \geq 2\omega/7$.*

According to the number of satisfied requests by the optimal offline algorithm, we classify the cells into two types: cell c_i is $safe$ if $O_i \leq 2\omega/3$, and *dangerous* otherwise.

Lemma 2 *If C_i is safe, then $A_i \geq 3O_i/7$*

Proof. This lemma can be proved by analyzing the following two cases.

- If $R_i \leq 2\omega/7$, $A_i = R_i \geq O_i$, then $A_i \geq 3O_i/7$.
- If $R_i > 2\omega/7$, CACO will accept at least $2\omega/7$ requests by assigning frequencies from F_x, thus, $A_i \geq 2\omega/7$. Since C_i is safe, $O_i \leq 2\omega/3$, thus, $A_i \geq 3O_i/7$.

□

Fact 3 *A safe cell has at most 3 dangerous neighboring cells. All neighboring cells around a dangerous cell are safe.*

Proof. This fact can be proved by contradiction. If a safe cell C has more than 3 dangerous neighboring cells, since C has 6 neighboring cells, there must exist two dangerous cells which are neighbors. From the definition of dangerous cell, the total number of accepted request in these two dangerous neighboring cells is strictly more than ω, contradiction!

Similarly, if a dangerous cell C' is a neighboring cell of another dangerous cell C, the total number of accepted request in C and C' is strictly more than ω. Contradiction! □

According to the algorithm, when a request cannot be satisfied in a cell C with color c, all frequencies in F_c must be used in C, and all frequencies in F_S must be used in C and its six neighbors. Thus, we have the following fact:

Fact 4 *If cell C cannot satisfy any request according to the algorithm CACO , then $G_S(C) + \sum G_S(C_k) \geq \omega/7$, where C_k is the neighboring cell of C.*

To compare the number of satisfied requests in each cell with the optimal offline solution, we define B_i as follows.

$$B_i = \begin{cases} 3O_i/7 & \text{if } C_i \text{ is safe} \\ A_i + \sum(A_k - 3O_k/7)/3 & \text{if } C_i \text{ is dangerous } (C_k \text{ is the neighboring cell of } C_i) \end{cases}$$

Lemma 5 $\sum B_i \leq \sum A_i$.

Proof. According to Lemma 2, if C_i is safe, we have $A_i \geq 3O_i/7$. From Fact 3, we know there are at most three dangerous neighbors around C_i, thus, after counting $B_i = 3O_i/7$ frequencies in C_i, the remaining $A_i - 3O_i/7$ frequencies can compensate the frequencies in the dangerous neighboring cells, and each dangerous cell receives $(A_i - 3O_i/7)/3$ frequencies. From the definition of B_i, we can see that $\sum B_i \leq \sum A_i$. □

Theorem 1 *The asymptotic competitive ratio of algorithm CACO is at most 7/3.*

Proof. From the definition of O_i and B_i, we can say $O_i/B_i \leq 7/3$ for any cell leads to the correctness of this theorem. That is because $\sum O_i / \sum A_i \leq \sum O_i / \sum B_i \leq \max O_i/B_i$

If the cell is safe, i.e., $O_i \leq 2\omega/3$, we have $O_i/B_i = 7/3$.

If the cell C_i is dangerous, i.e., $O_i > 2\omega/3$, since $R_i \geq O_i > 2\omega/3 > 3\omega/7$, that means the number of requests R_i in this cell is larger than A_i. Thus, some requests are rejected in cell C_i, moreover, this cell cannot accept any further requests.

- If the number of accepted requests in any neighbor of C_i is no more than $2\omega/7$, we can say that all the shared frequencies in F_S are assigned to requests in cell C_i. Thus, $A_i = 3\omega/7$. We have

$$O_i/B_i = O_i/(A_i + (\sum(A_k - 3O_k/7))/3) \leq O_i/A_i \leq \omega/A_i = 7/3.$$

- Otherwise, suppose there are m neighbors of C_i in which the number of accepted requests are more than $2\omega/7$. Let $\hat{O}_i$ denote the average number of optimum value of accepted requests for these m neighboring cells around

C_i.

$$B_i = 2\omega/7 + G_S(C_i) + (\sum_{\text{for all safe neighbors}} (A_k - 3O_k/7))/3$$
$$\geq 2\omega/7 + G_S(C_i) + (m \times 2\omega/7 + \sum_{\text{for the neighbors with } A_k > 2\omega/7} G_S(C_k) - m \times 3\hat{O}_i/7)/3$$
$$\geq 2\omega/7 + (m \times 2\omega/7 + \sum_{\text{for the neighbors with } A_k > 2\omega/7} G_S(C_k) + G_S(C_i) - m \times 3\hat{O}_i/7)/3$$
$$= 2\omega/7 + (m \times 2\omega/7 + \omega/7 - m \times 3\hat{O}_i/7)/3$$
$$\geq 2\omega/7 + (2\omega/7 + \omega/7 - 3\hat{O}_i/7)/3$$
(that is because for any neighbor with $A_k > 2w/7$,
$O_k \leq (\omega - O_i) \leq \omega/3, \text{thus}, \hat{O}_i \leq \omega/3$ and $2\omega/7 - 3\hat{O}_i/7 \geq 0$.)
$$\geq 2\omega/7 + (3\omega/7 - 3(\omega - O_i)/7)/3$$
(since $O_k \leq \omega - O_i$ leads to $\hat{O}_i \leq \omega - O_i$)
$$= 2\omega/7 + O_i/7$$

Thus, $O_i/B_i \leq O_i/(2w/7 + O_i/7) \leq 7/3$. □

In this kind of algorithms, the frequencies are partitioned into F_R, F_G, F_B and F_S, when a request arrives at a cell with color c, first choose the frequency from the set F_c, then from F_S if no interference appear. The performances are different w.r.t. the ratio between $|F_R|$ ($|F_G|$, $|F_B|$) and $|F_S|$. Note that from symmetry, the size of F_R, F_G and F_B should be same. Now we show that CACO is best possible among such kind of algorithms. Suppose the ratio between $|F_R|$ and $|F_S|$ is $x : y$. Consider the configuration shown in Figure 2. In the first step, ω requests arrive at the center cell C with color c, the algorithm will use up all frequencies in F_c and F_S, in this case, the ratio of accepted requests by the optimal offline algorithm and the online algorithm is $(3x + y)/(x + y)$ since the optimal algorithm will accept all these requests. In the second step, ω requests arrive at C_1, C_2 and C_3 with the same color c'. The online algorithm can only accept $x\omega/(3x + y)$ requests in each C_i $(1 \leq i \leq 3)$ since the frequencies in F_S are all used in C. In this case, the ratio between the optimal offline algorithm and the online algorithm is $3(3x + y)/(4x + y)$ since the optimal algorithm will accept all ω requests in C_i $(1 \leq i \leq 3)$ and reject all requests in C. Balancing these two ratios, we have $x : y = 2 : 1$, and the ratio is at least $7/3$.

3 Call Control in Triangle-Free Cellular Networks

The call control problem in cellular network is hard. But for some various graph classes, this problem may have a better performance. For example, in linear network, an optimal online algorithm with competitive ratio 3/2 can be achieved [10]. An interesting induced network, *triangle-free cellular network*, has been studied for many problems including frequency assignment problem[4, 9].

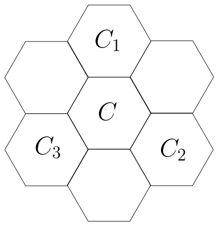


Fig. 2. Algorithm CACO is best possible among this kind of algorithms

For a given cell C_i, from the definition of triangle-free, only two possible configurations may exist for the structure of neighboring cells, which are shown in Fig. 3. It is easy to see that if C_i has 3 neighbors, the neighboring vertices are of the same color. On the other hand, if the neighbors are of different colors, C_i has at most 2 neighbors. There exists a simple structure in triangle-free cellular network, i.e., a cell has only one neighbor, we can regard this structure as the case in Fig. 3(b).

(a) Structure A: neighbors with the same base color

(b) Structure B: neighbors with different base colors

Fig. 3. Structure of neighboring cells

For the three base colors R, G and B, we define a cyclic order among them as $R \to G$, $G \to B$ and $B \to R$. Partition the frequency set $\{1, ..., \omega\}$ into three disjoint sets:

$$F_R = \{1, ..., \omega/3\},\ F_G = \{\omega/3 + 1, ..., 2\omega/3\},\ F_B = \{2\omega/3 + 1, ..., \omega\}$$

To be precisely, assigning frequencies from a set must in order of *bottom-to-top* (assigning frequencies from lower number to higher number) or *top-to-bottom* (assigning frequencies from higher number to lower number). Now we describe our algorithm for call control problem in triangle-free cellular networks.

Algorithm CACO2 : Handling request in a cell C with color $X \in \{R, G, B\}$

1. If cell C has no neighbors, just assign frequency from 1 to ω.

2. If cell C has neighboring structure A (Fig. 3(a)), let Y be the base color of C's neighbors and Z be the other third color. Assign frequency in cell C as follows if no interference appear:
   (a) Assign frequencies from F_X in bottom-to-top order.
   (b) If all frequencies in F_X are used up, assign frequencies from F_Z in bottom-to-top order if $X \to Y$; and in top-to-bottom order otherwise. Such assignment can make sure if C uses the frequency from F_Z after using up all frequencies from F_X, and its neighboring cell C' also uses the frequency from F_Z after using up the frequencies from F_Y, C and C' must assign frequency from F_Z in different order no matter the neighbor configuration of C' is. (This can be verified by checking this case (case 2) and the next case (case 3) of CACO2.)
3. If cell C has neighboring configuration B (Fig. 3(b)), let Y and Z be the base colors of its two neighbors, respectively. Without loss of generality, assume $X \to Y$. Assign frequency in cell C as follows if no interference appear:
   (a) Assign frequencies from F_X in bottom-to-top order.
   (b) If all frequencies in F_X are used up, assign frequencies from F_Y in top-to-bottom order.

Theorem 2 *The competitive ratio of CACO2 is at most 9/4.*

Proof. Assume at some time, let O_i and A_i denote the number of accepted requests in cell C_i by the optimal offline algorithm and online algorithm CACO2, respectively. The theorem holds if $\sum O_i / \sum A_i \leq 9/4$. Similar to the analysis for CACO, we define B_i as the amortized number of accepted requests in cell C_i. Thus, our target is to prove that $O_i/B_i \leq 9/4$ and $\sum B_i \leq \sum A_i$. W.l.o.g., let X, Y and Z denote the three colors in the network.

Intuitively, we may set $B_i = 4O_i/9$ if $A_i \geq 4O_i/9$ in cell C_i, and the remaining uncounted frequencies can be used to compensate the number of accepted frequencies in its neighboring cells. Next, we describe how to partition the remaining uncounted frequencies according to cell C_i's neighboring configuration. Let H_{ij} to be the number of frequencies used in C_i but will compensate the number of frequencies in C_j.

1. The neighboring configuration of C_i is A (Fig. 3(a)), the uncounted number of frequencies is $A_i - 4O_i/9$, evenly distribute this number to the three neighboring cells, i.e., each neighboring cell C_j of C_i receives $H_{ij} = (A_i - 4O_i/9)/3$.
2. The neighboring configuration of C_i is B (Fig. 3(b)), denote the color of C_i to be X, and the colors of its neighboring cells to be Y (cell C_j) and Z (cell C_k) respectively. W.l.o.g., assume $X \to Y$, $Y \to Z$ and $Z \to X$.
   – If $A_i > \omega/3$,
     In this case, the requests in cell C_i will use some frequencies from F_Y. If $A_j < 4O_j/9$, there exist rejected request in C_j, thus, $A_i + A_j = 2\omega/3$. The remaining uncounted number of frequencies in C_i can be partitioned into $(4O_j/9 - A_j)$ and $\omega/9$, the former part $(4O_j/9 - A_j)$ compensates

the number in C_j (i.e., $H_{ij} = 4O_j/9 - A_j$ if $A_j < 4O_j/9$) and the later part $\omega/9$ compensates the number in C_k (i.e., $H_{ik} = \omega/9$ if $A_k < 4O_k/9$). This compensation is justified since $4O_i/9 + (4O_j/9 - A_j) + \omega/9 = 4(O_i + O_j)/9 - A_j + \omega/9 \le 5\omega/9 - A_j < A_i$.

– If $A_i \le \omega/3$,
In this case, all frequencies used in C_i are from F_X, and some frequencies used in C_k may from F_X too. If $A_k < 4O_k/9$, all remaining uncounted number $A_i - 4O_i/9$ of frequencies in C_i will compensate the number in C_k, i.e., $H_{ik} = A_i - 4O_i/9$ and no extra number of frequencies compensates the number of frequencies in C_j, i.e., $H_{ij} = 0$.

Next, we define B_i as follows,

$$B_i = \begin{cases} 4O_i/9 & \text{if } A_i \ge 4O_i/9 \\ A_i + \sum H_{ji} & \text{if } A_i < 4O_i/9, \text{ where } H_{ji} \text{ is the compensation from neighboring cell } C_j \end{cases}$$

From previous description, we can say that $4O_i/9 + \sum_j H_{ij} \le A_i$ if $A_i \ge 4O_i/9$, thus,

$$\begin{aligned} \sum B_i &= \sum_{A_i \ge 4O_i/9} 4O_i/9 + \sum_{A_i < 4O_i/9} (A_i + \sum_{C_i \text{ and } C_j \text{ are neighbors}} H_{ji}) \\ &= \sum_{A_i \ge 4O_i/9} (4O_i/9 + \sum_{C_i \text{ and } C_j \text{ are neighbors}} H_{ij}) + \sum_{A_i < 4O_i/9} A_i \\ &\le \sum_{A_i \ge 4O_i/9} A_i + \sum_{A_i < 4O_i/9} A_i \\ &\le \sum_i A_i \end{aligned}$$

Now we analyze the relationship between B_i and O_i. Assuming the color of C_i is X.

1. If $A_i \ge 4O_i/9$, $B_i = 4O_i/9$.
2. If $A_i < 4O_i/9$,
   (a) If $A_i < \omega/3$
   Since $A_i < 4O_i/9$, there must exist some rejected requests in C_i. Some frequencies in F_X are used in one of C_i's neighbor C_j. According to the algorithm, the neighboring structure of C_j is B (Fig. 3(b)), and $A_i + A_j = 2\omega/3$.
   In this case, $H_{ji} = 4O_i/9 - A_i$, thus,
   $$B_i = A_i + \sum_{C_k \text{ and } C_i \text{ are neighboring cells}} H_{ki} \ge A_i + H_{ji} = 4O_i/9.$$
   (b) If $A_i \ge \omega/3$ and C_i has two neighbors C_j with color Y and C_k with color Z as shown in Fig. 3(b). W.l.o.g., assume that $X \to Y$, $Y \to Z$ and $Z \to X$. According to the algorithm, after using up the frequencies in F_X, C_i will use some frequencies from F_Y until interference appear, thus, $A_i + A_j \ge 2\omega/3$.

i. If the neighboring configuration around C_j is A (Fig. 3(a)), we claim that $A_j \geq 4O_j/9$. That is because $O_j \leq \omega - O_i \leq \omega - 9A_i/4 \leq \omega - 9\omega/12 = \omega/4$, $A_i \leq 4O_i/9 \leq 4\omega/9$, and $A_i + A_j \geq 2\omega/3$.
In this case, $H_{ji} = (A_j - 4O_j/9)/3$, and

$$B_i \geq A_i + H_{ji} = A_i + (A_j - 4O_j/9)/3 \geq 4O_i/9.$$

ii. If the neighboring configuration around C_j is B (Fig. 3(b)),
– if $A_j \leq \omega/3$, we have $H_{ji} = A_j - 4O_j/9$. Thus,

$$B_i \geq A_i + H_{ji} = A_i + A_j - 4O_j/9 \geq 2\omega/3 - 4O_j/9 \geq 4O_i/9.$$

– If $A_j \geq \omega/3$, $H_{ji} = \omega/9$, thus,

$$B_i \geq A_i + H_{ji} = A_i + \omega/9 \geq \omega/3 + \omega/9 = 4\omega/9 \geq 4O_i/9.$$

(c) If $A_i \geq \omega/3$ and the neighbors of C_i are of the same color (Fig. 3(a)), assume the color of its neighboring cell is Y. According to the algorithm, after using up the frequencies from F_X, C_i will use some frequencies from F_Z to satisfy some requests. Since C_i rejects some requests, we have $A_i + A_j = \omega$ for some neighboring cell C_j of C_i. This is because C_i and C_j assign frequencies from F_Z in different order, and C_j will use the frequency from F_Z after using up the frequency from F_Y.
In this case, $H_{ji} = (A_j - 4O_j/9)/3$ if the neighboring configuration of C_j is A (Fig. 3(a)), or $H_{ji} = \omega/9$ if the neighboring configuration of C_j is B (Fig. 3(b)). In the former case,

$$B_i \geq A_i + H_{ji} = A_i + (A_j - 4O_j/9)/3 > 4O_i/9;$$

in the later case,

$$B_i \geq A_i + H_{ji} = A_i + \omega/9 \geq 4\omega/9 \geq 4O_i/9.$$

Combine all above cases, we have $O_i/B_i \leq 9/4$ in each cell C_i. Since $\sum B_i \leq \sum A_i$, we have $\sum O_i / \sum A_i \leq 9/4$.

Next, we show that the lower bound of competitive ratio for call control problem in triangle-free cellular networks is at least 5/3.

Theorem 3 *The competitive ratio for call control problem in triangle-free cellular network is at least 5/3.*

Proof. We prove the lower bound by using an adversary who sends request according to the assignment of the online algorithm.

Consider the configuration shown in Figure 4.

In the first step, the adversary sends ω requests in the center cell C. Suppose the online algorithm accepts x requests. If $x \leq 3\omega/5$, the adversary stop sending request. In this case, the optimal offline algorithm can accept all these ω requests, thus, the ratio is at least 5/3.

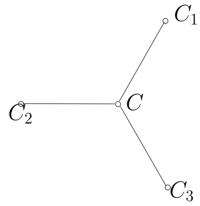


Fig. 4. lower bound of competitive ratio is at least 5/3

If $x > 3\omega/5$, the adversary then sends ω requests in each cell of C_1, C_2 and C_3. To avoid interference, the online algorithm can accept at most $\omega - x$ requests in each cell, and the total number of accepted requests is $x + 3(\omega - x) = 3\omega - 2x$. In this case, the optimal offline algorithm will accept 3ω requests, i.e., reject all requests in the center cell C. Thus, the ratio in this case is $3\omega/(3\omega - 2x)$. Since $x > 3\omega/5$, this value is at least 5/3.

Combine the above two cases, we can say that the competitive ratio is at least 5/3. □

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