Department of Computer Science, The University of Hong Kong

CS Seminar

Understanding Quaternions

Prof. Ron Goldman
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Rice University

Date:
May 31, 2010
Monday
3:00 pm

Venue:
Room 308
Chow Yei Ching Building
The University of Hong Kong

Abstract:

Quaternions are vectors in 4-dimensions endowed with a rule for multiplication that is associative but not commutative, distributes through addition, and has an identity and inverses. Thus the quaternions form a division algebra. Ever since the discovery of quaternions, quaternion multiplication has been used to rotate vectors in 3-dimensions by sandwiching a vector between a unit quaternion and its conjugate.

In Computer Graphics quaternions have three principal applications: to increase speed and reduce memory for calculations involving rotations; to avoid distortions arising from numerical inaccuracies caused by floating point computations with rotations; and to interpolate between two rotations for key frame animation.

Yet while the formal algebra of quaternions -- multiplication, sandwiching, interpolation – is well established in Computer Graphics, the geometry of quaternions is not well understood. The formulas for multiplication and sandwiching work, but it is hard to see how anyone ever came up with these formulas. Even the geometric meaning of a quaternion -- a scalar added to a vector – is an enigma. The purpose of these lectures is to develop a better intuitive understanding of quaternions, and to remove much of the mystery surrounding these formulas.

The main goals of these lectures are to make five principal contributions:
1. To provide a fresh, geometric interpretation for quaternions, appropriate to contemporary Computer Graphics;
2. To present better ways to visualize quaternions, and the effect of quaternion multiplication on points and vectors in 3-dimensions based on insights from the algebra and geometry of multiplication in the complex plane;
3. To derive the formula for quaternion multiplication from first principles;
4. To develop simple, intuitive proofs of the sandwiching formulas for rotation and reflection;
5. To show how to apply sandwiching to compute perspective projections.

In addition to these theoretical issues, we shall also address some computational questions. Our objectives here are to:
1. Develop straightforward formulas for converting between quaternion and matrix representations for rotations, reflections, and perspective projections;
2. Discuss the relative merits of the quaternion and matrix representations for these three transformations;
3. Explain how to avoid distortions due to floating point computations with rotations by using unit quaternions to represent rotations;
4. Derive the formula for spherical linear interpolation (SLERP), and explain how to apply this formula to interpolate between two rotations for key frame animation.

About the Speaker:

Ron Goldman is a Professor of Computer Science at Rice University in Houston, Texas. Professor Goldman received his B.S. in Mathematics from the Massachusetts Institute of Technology in 1968 and his M.A. and Ph.D. in Mathematics from Johns Hopkins University in 1973. He is an associate editor of Computer-Aided Design and Computer Aided Geometric Design. In 2002, he published a book on Pyramid Algorithms: A Dynamic Programming Approach to Curves and Surfaces for Geometric Modeling.

Dr. Goldman's research interests lie in the mathematical representation, manipulation, and analysis of shape using computers. His work includes research in computer aided geometric design, solid modeling, computer graphics, and splines. He is particularly interested in algorithms for polynomial and piecewise polynomial curves and surfaces, and he is currently investigating applications of algebraic and differential geometry to geometric modeling.

All are welcome!
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